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**COMMENT**

# Comment on 'Generalized commutators and deformation of strong-coupling superconductivity'

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**Abstract.** It is shown that a commonly used deformation of the fermionic canonical anticommutation relations is equivalent to the undeformed relations.

Chen and Ho [1] discuss a strong-coupling model of superconductivity based on a microscopic hamiltonian expressed in terms of fermionic operators satisfying

$$\begin{aligned} &aa^\dagger + qa^\dagger a = q^N \\ &[N, a^\dagger] = a^\dagger \\ &a^2 = 0. \end{aligned} \tag{1}$$

They refer to these as fermionic $q$-oscillators.

In fact, equation (1) is equivalent to the usual anticommutation relation

$$\begin{aligned} aa^\dagger + a^\dagger a &= 1 \\ [N, a^\dagger] &= a^\dagger \\ a^2 &= 0 \end{aligned} \tag{2}$$

and describes ordinary fermions.

This equivalence may be seen as follows:
(A) *The (usual) fermionic operators $a$, $a^\dagger$ satisfying equation (2) also satisfy equation (1).* It is easy to show that here $N \equiv a^\dagger a$ is idempotent, so

$$e^{sN} = 1 + N(e^s - 1). \tag{3}$$

Setting $q \equiv e^s$, we may rewrite equation (3) as

$$aa^\dagger + qa^\dagger a = q^N.$$

(B) Operators $a$, $a^\dagger$ satisfying equation (1) also satisfy equation (2). From equation (1) it follows (and the authors note) that the eigenvalues of $N$ are 0 and 1. Therefore $[N] = N$ (on number states, which form a basis) where

$$[N] = \frac{q^N - q^{-N}}{q - q^{-1}}$$

and

$$\begin{aligned} a^\dagger a &= [N] = N \\ aa^\dagger &= [1 - N] = 1 - N \end{aligned}$$

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which gives

$$aa^\dagger + a^\dagger a = 1.$$

The authors' assumption that $a^2 = 0$ is not strictly necessary. If, assuming equation (*1*), we attempt to build up $N$ eigenstates from the vacuum $|0\rangle$ defined by $a|0\rangle = 0$, we see that $|2\rangle = a^\dagger|1\rangle$ is not normalizable, so there is only $\{0, 1\}$ occupancy.

## References

[1] Chen Wei-Yeu and Ho Choon-Lin 1993 *J. Phys. A: Math. Gen.* 26 4827–33